7. Interference Temperature Analysis for Fixed Point-to-Point Services

7.1 Introductory comments on fixed point-to-point services

The FCC has established several fixed point-to-point wireless services for microwave to millimeter-wave radio spectrum. Some of the more familiar services located in spectrum below 15 GHz include:

- Fixed Microwave Services (FS), including Private Operational Fixed Point-to-Point Microwave Service (POFS) and Common Carrier Fixed Point-to-Point Microwave Service under Part 101, in 2 GHz (now mostly relocated), 4 GHz, 6 GHz and 10.5 GHz frequency bands;
- Domestic Public Fixed Radio Services, the International Fixed Public Radiocommunication Services, and the Fixed Microwave Services under Parts 21, 23, and 101, respectively (typically provided by, among others, industrial, governmental and transportation related licensees), in the 6525-6700 MHz band; and
- Broadcast Auxiliary Systems (BAS) under Part 74, cable TV relay systems (CARS) under Part 78, and fixed microwave systems under Part 101 in the 12.7 to 13.2 GHz frequency band.

The NPRM seeks comment on a proposal to implement ITemp in these last two bands.

These fixed service links may carry one-way or two-way (frequency-duplexed) traffic, and may employ multiple channels for high capacity needs. The relatively high frequencies and wide signal bandwidths utilized necessitate that there be line-of-sight clearance between transmit and receive antennas along the length of the link. The use of highly directional antennas, mounted well above ground level on towers or rooftops of high buildings, further illustrate the importance of uncluttered propagation paths. The fact that highly directional antennas have high on-boresight gain also help to reduce the transmit power required to complete the power budget for any given link geometry.

Outage definitions for FS point-to-point links have historically been made based on the type of traffic carried on the link [1]. For telephony circuits, the measure of concern is worst-circuit noise levels. Links carrying video traffic are subject to a wide variety of distortion criteria, such as waveform distortion, chrominance-luminance gain/delay inequality, chrominance-to-luminance intermodulation, weighted random noise, and impulse noise. A general treatment for digital systems may introduce a maximum bit error rate threshold (e.g. 10^{-6}) for 95% of connections in a month, or a minimum required proportion (typically on the order of 99.5%) of error-free seconds over a measurement interval.

For digital modems used on FS point-to-point links, a considerable body of research and engineering practice exists around the use of parameterized, statistically-characterized multipath models to predict link outage probabilities [2]. The frequency of operation, link path length and terrain classification, type of modem employed (*e.g.*, what combination of modulation and coding are employed), known co-channel interference and receiver SINR requirements all factor into outage calculations. Analog modems in CARS bands are to first order impacted by signal power attenuation due to atmospheric moisture. Signal attenuation in rainfall increases with frequency of operation and with rainfall intensity (precipitation rate in in/hr or mm/hr), and relationships between rainfall intensity and signal power loss per unit distance of precipitation-affected path as a function of frequency have been developed based on analysis and experimental data. Statistical models for rainfall rate have been similarly developed for various geographic regions, which permit outage probability estimates for analog links employed in these regions [3].

FS link budgets are calculated in order to provide high availability of the supported service, which generally requires conservative margins to account for the key statistically-characterized variable impairments on the link. To ensure that such margins adequately protect the link, it is important to accurately characterize other fixed and variable link impairments, such as noise and co-channel interference. If the magnitude of other impairments is underestimated, then actual availability will be less than predicted or calculated. This is the pernicious effect of an increase in "background" interference that might be observed with the introduction of spectrum sharing between licensed FS point-to-point links and low-power unlicensed devices. Given conservative engineering margins and high link availability requirements, a FS point-to-point link will almost always be able to tolerate interference from underlaid unlicensed devices, so it might appear that sharing can be easily and profitably accommodated. However, if a higher-than-anticipated level of background interference is always present, then a less severe degree of impairment (e.g., a smaller degree of amplitude dispersion across the channel bandwidth, or a smaller amount of rain attenuation of signal power) from the key physical phenomena around which the link was engineered will be required to induce outage. The probability of occurrence of this less severe degree of impairment will always be greater than a more severe degree, so <u>the effect of an unanticipated increase in background interference is an increase in the outage time, or equivalently a decrease in service availability for the affected link</u>. If the supported service cannot tolerate this decrease in availability, then engineering mitigations such as diversity, antenna improvements, or increased transmit power – all of which impose significant cost to the link operator – must be employed

The remainder of this section addresses the introduction of unlicensed device underlays to the frequency bands used by FS point-to-point links. The FCC has proposed that this sharing of spectrum be accomplished through a specific, simplified take on the ITemp concept. The methods recommended by the FCC are carefully analyzed and quantified, with particular reference to operating parameters for realistic FS links. Alternatives to this simplified interference temperature approach for spectrum sharing are also evaluated.

7.2 Explicit consideration in NPRM

In discussion of general ITemp implementation approaches, the FCC considered a feedback approach based on the victim receiver's view of interference. The suggestion is for the receive sites to measure the "temperature" and forward the information to a central site, where the ITemp profile for an area could be computed, and then messages could be broadcast to unlicensed devices to communicate the current ITemp status and possibly inhibit their transmission if it is too high. The FCC further indicates that "this scenario may be appropriate in services such as those involving fixed point-to-point operations where there are relatively few receive sites in a given area."[8]

The feedback principle is intuitively sensible, and, as discussed above, is required if it is not possible to define conservative engineering rules for exclusion zones that can accommodate any reasonable density of underlaid unlicensed devices. However, the idea of an ITemp "profile" for an area should not be employed to somehow average out the incidence of a single receiver that may be experiencing interference at or near its total interference threshold. Also, there are some practical obstacles to the implementation of a feedback mechanism for ITemp-based sharing with FS microwave systems. These will be addressed later in the chapter.

7.2.1 Initial FCC Analysis Supporting Application of Dynamic Frequency Selection (DFS) and Transmit Power Control (TPC)

Though the feedback principle is mentioned in conjunction with spectrum sharing between fixed service point-to-point links and unlicensed devices, the FCC suggests that it may be possible to share spectrum when the unlicensed devices use dynamic frequency selection (DFS) and transmit power control (TPC), based on measurements of the licensed transmitter, without any feedback from the licensed (victim) receiver.[9]

Several key assumptions are made in the development of an analysis sketch to establish this point. First, the FCC implies that the worst case interference will occur when the unlicensed device is located close to the receiver.[10] Second, the FCC states that "the unwanted emissions received by the FS receiver will be dominated by the emissions from the closest device."[11] Restated, this assumption suggests that the FCC believes that interference aggregation effects will not be of significant consequence. Third, the FCC assumes that the unlicensed device can assess its interference impact on the licensed receiver by measuring the power it receives from the licensed transmitter.[12] This third assumption provides a rationale for a transmit power control (TPC) scheme in which the unlicensed device transmits at a power level equal to the power level that it receives from the licensed transmitter, plus a constant offset that takes into account (i) receive S/I requirements of the licensed service; (ii) the minimum path loss between the ground-level unlicensed device and the elevated, tower-mounted FS receive antenna; and (iii) the off-boresight discrimination of the directional FS receive antenna, which will suppress the interference impact of the unlicensed device by a large amount (typically > 30 dB) relative to the gain it offers to the FS signal transmitted by the licensed paired node. The TPC equation for a single unlicensed interferer can be written as

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[8] NOI/NPRM, at ¶11.
[9] *Id*, at ¶40-46.
[10] *Id*, at ¶40-41.
[11] *Id*, at ¶41.
[12] *Ibid*, at ¶43.

$$P_{TX;UL} = P_{RX;L,UL} + X \tag{145}$$

where $P_{TX;UL}$ is the unlicensed transmitter's power into its antenna, $P_{RX;L,UL}$ is the power from the licensed transmitter as measured by the unlicensed device,[13] and X is a parameter whose value is to be determined. All quantities are assumed to be log-transformed to decibel values so that losses and gains are additive rather than multiplicative.

Now to develop expressions for the parameter X, let us first consider the power measured by the unlicensed device:

$$P_{RX;L,UL} = P_{TX;L} + G_{TX;L,UL} - L_{L,UL} + G_{RX;L,UL} \tag{146}$$

In words, the licensed transmit power measured by the unlicensed device equals the power into the licensed transmitter's antenna, plus the licensed transmitter's antenna gain in the direction of the interferer, minus the path loss to the interferer, plus the gain of the interferer's receive antenna.

Consider for now the effect of only a single interfering device. The interference power seen by the victim receiver is also expressed in straightforward fashion:

$$P_{RX;UL,L} = P_{TX;UL} + G_{TX;UL,L} - L_{UL,L} + G_{RX;UL,L} \tag{147}$$

Substituting (145) for $P_{TX;UL}$ into (yields

$$P_{RX;UL,L} = P_{TX;L} + G_{TX;L,UL} - L_{L,UL} + G_{RX;L,UL} + X + G_{TX;UL,L} - L_{UL,L} + G_{RX;UL,L} \tag{148}$$

and then substituting (146) into 147) gives

$$P_{RX;UL,L} = P_{TX;L} + G_{TX;L,UL} - L_{L,UL} + G_{RX;L,UL} + X + G_{TX;UL,L} - L_{UL,L} + G_{RX;UL,L} \tag{149}$$

To maintain the performance of the incumbent link in the presence of the unlicensed interferer, we must limit the interference into the victim receiver:

[13] The subscripting style "a, b" has the following embedded meaning. The first item ("a") indicates the transmitter (type), and the second term ("b") indicates the receiver (type) for a given path loss, antenna gain term (receive or transmit direction), or received power level. For example, "*L*, *UL*" implies "from the *licensed* transmitter to the *unlicensed* receiver." Such link orientation is not required to identify a transmit power level, however.

$$P_{RX;UL,L} \le P_{RX;L,L} - S \tag{150}$$

where $P_{RX;L,L}$ is the desired signal power seen at the victim receiver, and S is the required signal-to-impairment ratio in dB, which will include margins as determined by link budget calculations and consideration of propagation anomalies such as rain fading. For a given link, S is a fixed parameter calculated to provide a level of service (typically, outage probability).

The desired signal power is easily calculated from other quantities:

$$P_{RX;L,L} = P_{TX;L} + G_{TX;L,L} - L_{L,L} + G_{RX;L,L} \tag{151}$$

Now, we substitute (148) into the left-hand side of (150), and the expression for $P_{RX;L,L}$ into the right-hand side. Note that the licensed transmit power term $P_{TX;L}$ is common to both sides and cancels. Rearranging and collecting terms, we arrive at an inequality for the parameter X:

$$X \le \left[G_{TX;L,L} - G_{TX;L,UL}\right] + \left[G_{RX;L,L} - G_{RX;UL,L}\right] + \left[L_{L,UL} - L_{L,L}\right] + L_{UL,L} - \left[G_{RX;L,UL} + G_{TX;UL,L}\right] - S \tag{152}$$

This inequality provides an upper bound on X which decreases as the link target SINR S increases. This is correct, because an upper bound on X means that there is an upper bound on the power transmitted by the unlicensed device in order to limit its impact on the victim receiver.

The first term on the right-hand side of (152) is the difference in licensed transmitter antenna power gain as seen by the licensed receiver and the unlicensed receiver (which is making a power measurement). The NPRM correctly reasons that when the unlicensed interferer is near the victim receiver, this term is approximately zero, because the off-boresight angle to the interferer is approximately zero. Of course, when the unlicensed interferer is not near the victim receiver, then this is no longer true.

The second term is the difference in licensed (victim) receiver antenna power gain as seen by the licensed transmitter and the unlicensed interferer. The NPRM reasons that when the unlicensed interferer is near the victim receiver, this term will be at least 30 to 35 dB, because for reasonable victim receiver antenna heights, the off-boresight angle to the interferer is on the order of 20° or more, and FS antennas have minimum requirements on their off-angle boresight discrimination. Note however that this term can decrease significantly as the interferer distance from the victim receiver is increased, particularly if the interferer moves at ground level along the centerline of the FS link.

The third term is the difference in (free space) path loss from the licensed transmitter to the licensed receiver and to the unlicensed receiver. The NPRM correctly reasons that when the unlicensed interferer is near the victim receiver, this term is approximately zero, because the path lengths are nearly identical.

The fourth term is the path loss from the unlicensed interferer to the victim receiver. The NPRM notes that at the high frequencies used for these FS links, the path loss is very large for even modest interferer-to-victim separations, such as the elevation difference between ground level and the receiver antenna mounting.

The fifth term is the net gain introduced by the unlicensed interferer's antenna. Note that this term subtracts from X, because antenna gain both boosts the interferer's sense of the licensed transmitter power that it measures, and the amount of interference power it provides to the victim receiver. Normally this term will be small compared to the others, and in fact is not specifically mentioned in the NPRM analysis. It is included here for completeness.

It will be helpful to lump these five terms together into a single term F:

$$X \leq F - S \tag{153}$$

which then depends only on the location of the interferer and the antenna characteristics. For any interferer location, the maximum value of X (and therefore the maximum allowable transmitted power) is realized when the inequality is satisfied with equality.

The NPRM shows a calculation for X that assumes the unlicensed interferer is close to the victim receiver, in the senses identified above. However, a general TPC rule that relies only on power measurements by the unlicensed device must also consider the possibility that X may vary over the potential deployment range of the unlicensed devices, because the individual terms that contribute to X certainly do vary, in ways that do not necessarily compensate each other.

This can be explained simply by rearranging (152):

$$\begin{aligned} X \leq & \left[\text{licensed TX antenna discrimination differential}\right] \\ & + \left[\text{licensed RX antenna discrimination differential}\right] \\ & - \left[\text{unlicensed antenna gain effect}\right] \\ & + \left[L_{L,UL} + L_{UL,L}\right] - L_{L,L} - S \end{aligned} \tag{154}$$

If all antennas were completely isotropic, then the only term that depends on the location of the unlicensed interferer is the sum of path losses, from licensed transmitter to unlicensed interferer, and from unlicensed interferer to licensed (victim) receiver. This is illustrated in Figure 36.

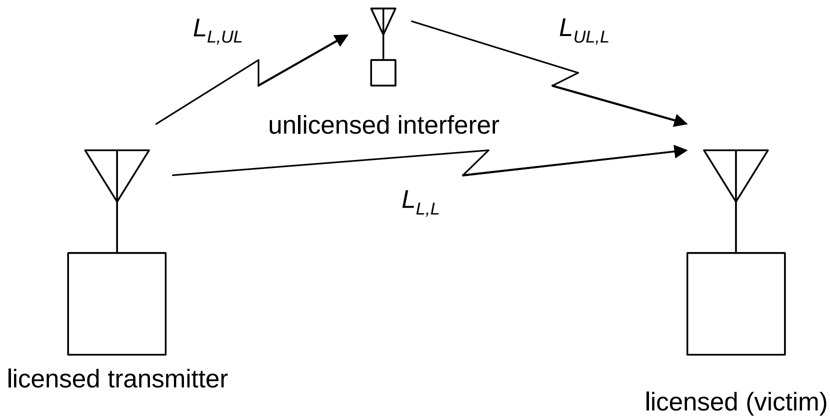

Figure 36 -- *Simple geometry for transmitter measurement-based TPC*

A straightforward geometric argument shows that the sum of path losses to and from the unlicensed interferer is minimized when the unlicensed interferer is at the center of the line segment connecting the licensed transmitter to the licensed receiver. In particular, it is obvious that the sum of the path lengths is minimized when the unlicensed interferer is anywhere on that line segment. Deviations away from the center line will result in longer combined path lengths, higher combined path losses, and higher values of parameter *X for that specific location.* Incorporation of antenna characteristics that also vary with location of the unlicensed interferer will add further complication. In the end, it will be necessary to choose a single value for the parameter *X* that guarantees that for a given probability of outage objective, the resulting interference to the victim receiver does not degrade its SINR to a value below *S*.

We can quantify this effect, for the single interferer case, by looking at a specific example based on parameters from an operational licensed FS link.

7.2.2 Specific Scenario Analysis

We develop an analytical treatment of unlicensed interference into licensed fixed point-to-point services around the simple geometric model of Figure 37.

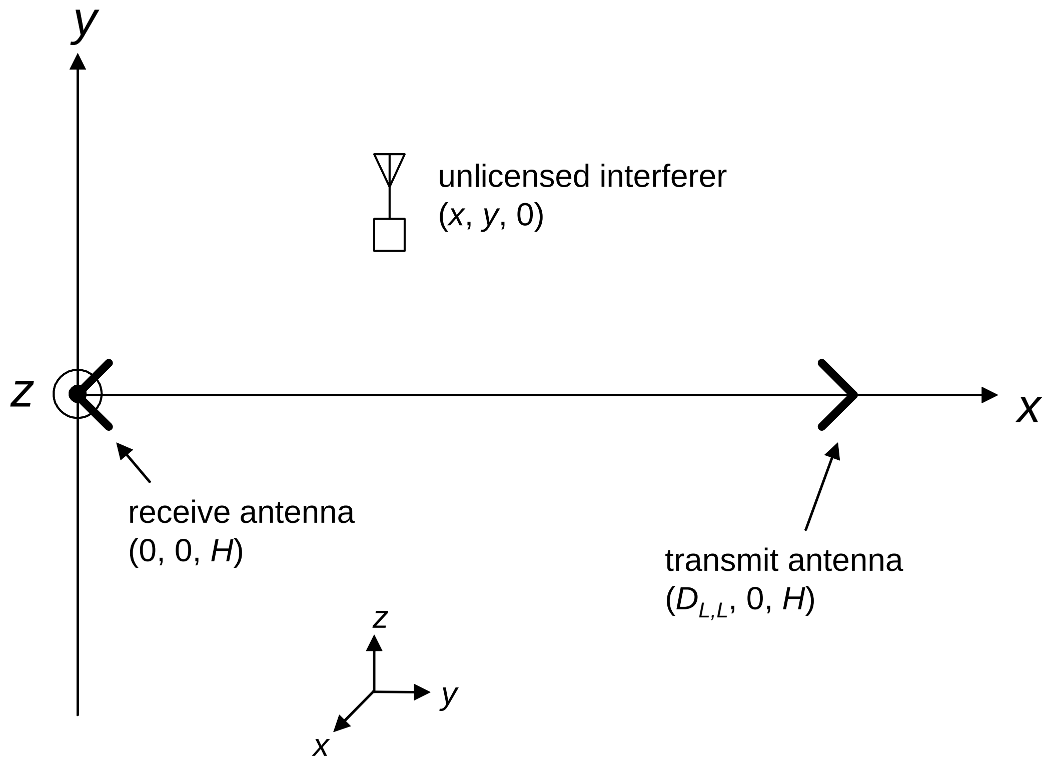

Figure 37 -- *Point-to-point interference geometry*

The interferer is assumed to be at ground level, and the microwave link's transmit and receive antennas are assumed to be at a common height *H* above ground level (AGL). The path length for the fixed microwave service is $D_{L,L}$ (the subscript implying "licensed-to-licensed"), and its transmitter and receiver antennas are assumed to be boresight-aligned. The FS link is assumed to employ vertically-polarized transmissions, and use antennas with circularly-symmetric power gain patterns about the boresight. The unlicensed interferer is assumed to use an antenna that has discrimination only in elevation angle, and is omnidirectional in azimuth (such as a vertical dipole).

From this simple model we can calculate various parameters that are relevant to the inequality for the parameter *X*. $L_{L,L}$ is calculated from $D_{L,L}$ by straightforward application of the free-space path loss equation

$$L_{L,L}\left(D_{L,L}\right) = 20\log\left(\frac{4pD_{L,L}}{l}\right) \tag{155}$$

where l is the carrier wavelength in meters. Similarly,

$$L_{L,UL}\left(D_{L,UL}\right) = 20\log\left(\frac{4pD_{L,UL}}{l}\right) \tag{156}$$

If a three-dimensional rectangular coordinate system is defined with the victim receiver at coordinates (0, 0, H), the licensed transmitter at ($D_{L,L}$, 0, H), and the unlicensed interferer at (x, y, 0), then

$$L_{UL,L}\left(D_{UL,L}\right) = 20\log\left(\frac{4pD_{UL,L}}{l}\right) \tag{157}$$

$$D_{UL,L} = \sqrt{x^2 + y^2 + H^2} \tag{158}$$

$$f_{L,UL} = \arccos\left(\frac{D_{L,L} - x}{D_{L,UL}}\right) \tag{159}$$

It is also straightforward to show that

$$f_{UL,L} = \arccos\left(\frac{x}{D_{UL,L}}\right) \tag{160}$$

$$e_{L,UL} = \arcsin\left(\frac{H}{D_{L,UL}}\right) \tag{161}$$

$$e_{L,UL} = \arcsin\left(\frac{H}{D_{L,UL}}\right) \tag{162}$$

and

$$e_{UL,L} = \arcsin\left(\frac{H}{D_{UL,L}}\right) \tag{163}$$

FCC license WLY-503 is for a fairly typical CARS link operated in western Florida by Sprint (Bay Area), Inc. Relevant technical parameters for this link are provided in Table 1.

Operating frequency	channel centers range from 12.7205 to 12.8115 GHz; assume 12.72 GHz
Transmitter location	28° 4’ 4” N latitude, 82° 24’ 56” W longitude
Receiver location	28° 2’ 20” N latitude, 82° 39’ 29” W longitude
Path length	23.996 km
Transmit power	+18.0 dBm (into antenna)
Transmit antenna height	265.0 ft AGL
Receive antenna height	200.0 ft AGL
Transmit antenna type	Parabolic, 8 ft diameter
Receive antenna type	Parabolic, 8 ft diameter

Table 1 -- *Technical parameters for example CARS license WLY-503*

We will simplify the geometric model slightly by using a flat-earth treatment, and by assuming a common antenna height of 70 m (about 230 ft), which together eliminate consideration of antenna downtilt to align transmit and receive boresights.

To model the properties of the circular parabolic reflector specified as the transmit and receive antenna types in Table 1, we employ radiation pattern envelope (RPE) data for a common commercially-available antenna with similar parameters. The RPE data models antenna discrimination from boresight as a piecewise-log-linear function of off-boresight angle; with the addition of the on-boresight antenna gain, we get a conservative envelope of the azimuth power gain pattern, as shown in Figure 38.

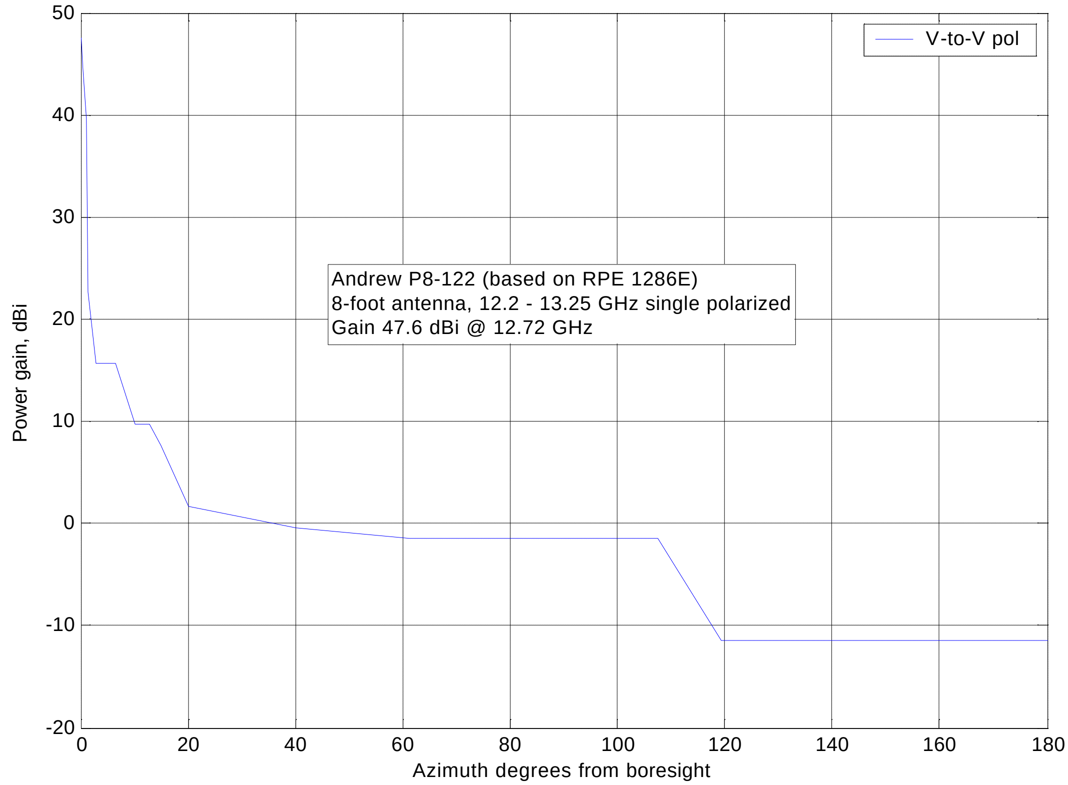

Figure 38 -- *Power gain envelope based on RPE data for 8 foot dish*

We will make the necessary assumption that the antenna power gain at any off-boresight solid angle is equal to the gain given by RPE data at the equivalent azimuth plane angle.[14]

To complete the numerical treatment, we will assume that the unlicensed interferer employs a half-wave dipole, oriented to the vertical. The power gain pattern of this antenna is known to be

$$G_{l/2}(e) = G_0 \left[\frac{\cos\left(\frac{p}{2}\sin e\right)}{\cos e} \right]^2 \tag{164}$$

as a function of elevation angle ***e***, and the maximum gain G_0 is 1.64 (2.15 dBi).

Let's first consider a case similar to what appears in the NPRM, which led to the suggestion that the parameter *X* could be on the order of 71 to 91 dB.[15] Suppose the

[14] The ideal parabolic reflector has a circularly symmetric pattern about the boresight.

interferer is located 100 meters from the base of the tower on which the receive antenna is mounted. Using reasoning similar to that employed in the NPRM, straightforward calculations show

- the victim receiver antenna's off-boresight angle to the interferer is at least 35° (when the interferer is on the centerline of the licensed link);
- therefore the discrimination of the victim receiver antenna in the direction of the interferer is at least 47.5 dB, according to Figure 38;
- the path loss from the interferer to the victim receiver antenna is 96.3 dB at 12.72 GHz; and
- the combined transmit and receive contributions of the unlicensed interferer's half-wave dipole amounts to approximately 1.9 dB.

Adding the previously-stated assumptions about transmit antenna gain and path loss differentials when the interferer is close to the victim receiver, we calculate that for this case,

$$F = 142.8\text{ dB}$$

$$X_{\max} = 142.8 - S$$

So if the victim receiver needs to maintain an overall SINR of 50 dB, then X must be no greater than 92.8 dB, which is more optimistic than the NPRM calculation. It is then straightforward to show that the interferer measures a power level of approximately -74.3 dBm from the licensed transmitter, and therefore it could transmit up to a power level of +18.5 dBm without harming the operation of the incumbent service.

Of course in practice, an interferer that is simply measuring power levels cannot ascertain its specific location with respect to the victim receiver. Suppose instead that we randomly locate the unlicensed interferer within a rectangle bounded by the lower left and upper right points (0, $-D_{L,L}$, 0) and ($D_{L,L}$, $D_{L,L}$, 0) (with respect to Figure 37). Is there much variation among the collected terms in the expression for X? For each unlicensed interferer location, we can calculate the term F and then look at how the values of F are distributed.

Figure 39 shows a distribution of F generated from 50,000 randomly selected points in the rectangle of area $2D_{L,L}{}^2$ (about 1,150 km^2). The resulting curve is fairly smooth. We notice that the value of F estimated for the near-to-victim case above, 142.8 dB, does in fact appear to be conservative, since only about 0.4% of unlicensed interferer locations have smaller F values (though up to 13.5 dB lower).

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[15] NOI/NPRM, at ¶43.

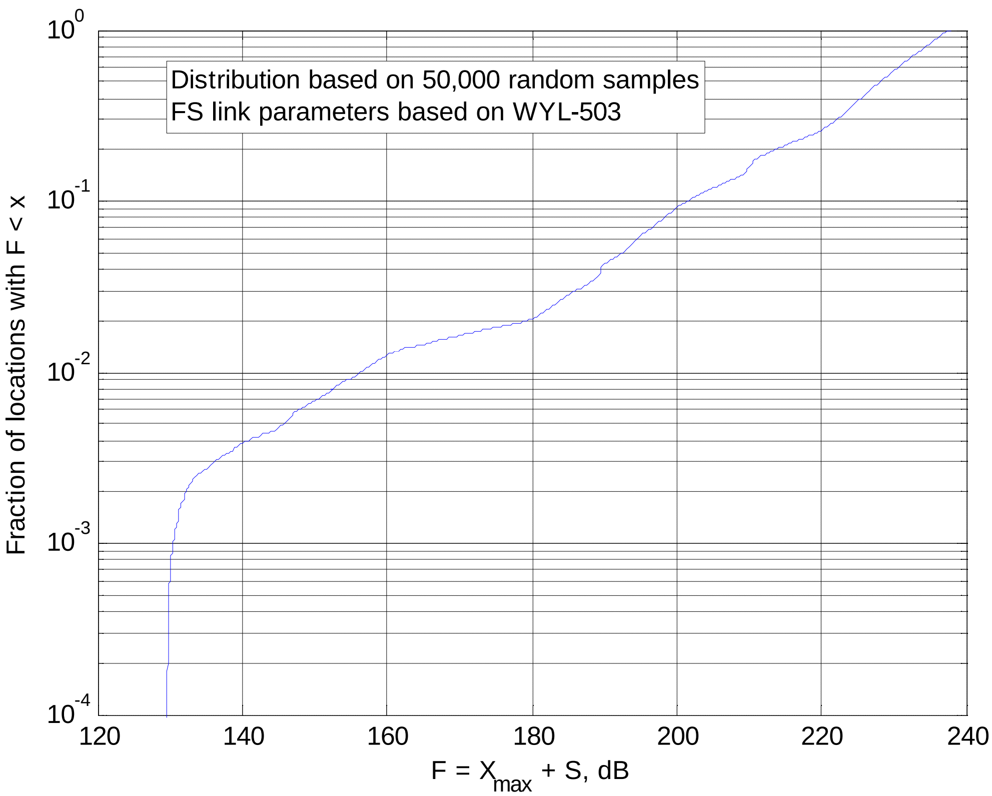

Figure 39 -- *Distribution of F for point-to-point FS scenario*

Indeed, the tails of the distribution are most relevant to choosing a TPC rule that keeps probability of link outage low. Since FS microwave links are typically engineered for well under 1% errored-seconds over long periods of service, and the link budget must already include margin for other variable impairments such as multipath and rain fading, reference to tail statistics is very important for selection of a value of the *X* parameter that would not significantly increase the outage time for the link.

If we consult the 0.5% tail level for the distribution of Figure 39, we get a corresponding *F*-value of 145.8 dB. A consequence of this assignment of the parameter *X* is summarized in Figure 40, where the distribution of unlicensed transmit power allowed under the TPC rule is plotted based on 50,000 random placements of the unlicensed device. The distribution assumes a minimum target SINR of 50 dB at the victim receiver (S = 50 dB, so X = 95.8 dB).

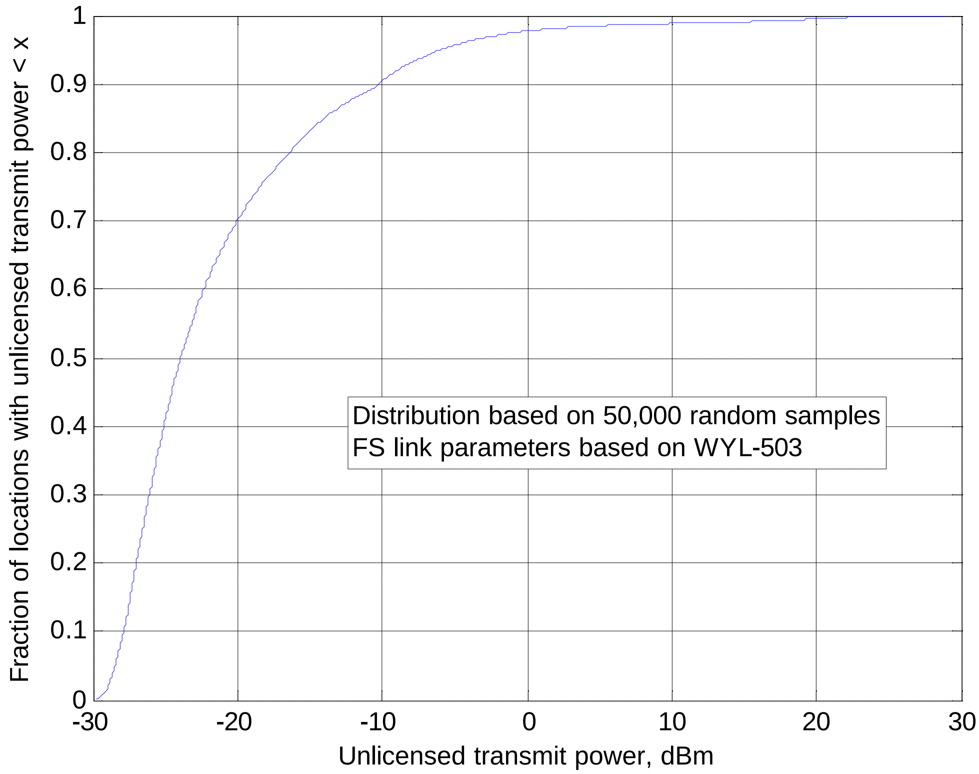

Figure 40 -- *Example distribution of unlicensed TX power for X = 95.8 dB*

From this plot, it is apparent that the single-parameter TPC rule allows for very limited unlicensed transmitter power, at least for the case where the victim receiver's target SINR is 50 dB. Fully 90% of locations permit no greater than -10 dBm (0.1 mW) of transmit power for a single interferering device. Conversely, only about 0.1% of locations allow an unlicensed transmit power level of 10 dBm (10 mW) or greater.

Note that this distribution does not take into account the limit to detection of receive power levels imposed by the noise floor of the unlicensed receiver. In fact, at the full signal bandwidth of the FS link (6 MHz), the thermal noise floor is at -106.1 dBm; with realistic unlicensed device electronics, the receiver noise figure will raise the noise floor by another 5 to 10 dB. With the use of a TPC rule parameter of X = 95.8 dB, it is clear that power levels below approximately -5 dBm cannot be established with this method. However, according to Figure 40, power levels higher than -5 dBm will cause harmful interference to the FS receiver at greater than 95% of all possible unlicensed transceiver locations in the study area. We can only conclude that even a seemingly conservative selection of TPC parameter X (based on tail statistics at the 0.5% level) for this single-interferer scenario will result in harmful interference to the FS receiver when the receiver noise performance of the unlicensed device is taken into account. Of course lower values of X will result in even lower levels of transmit power for the unlicensed devices.

This interference-driven transmit power analysis leads to a usage interpretation for the spectrum sharing arrangement. Suppose that the unlicensed device requires a minimum SINR of S_U dB for its transmission to be successfully recovered by a compatible unlicensed receiver. Except in cases where the unlicensed transmitter/receiver pair is located very near to the licensed transmitter (where the device-to-device separation might be significant compared to the FS transmitter-to-unlicensed device distance), they will see very comparable antenna gains, path losses, and therefore received power levels from the licensed transmitter. While this received power level establishes the unlicensed transmitter's power level, the licensed transmitter's signal is interference to the unlicensed receiver. The link budget (linear power basis) for the unlicensed receiver is readily stated:

$$P_{RX;UL,UL} = P_{TX;UL} \times G_{TX;UL,UL} \times G_{RX;UL,UL} \div L_{UL,UL} \tag{165}$$

and

$$P_{RX;UL,UL} \geq S_U \times \left(P_{RX;L,UL} + P_{\text{noise}}\right) \tag{166}$$

with

$$L_{UL,UL} \leq X \times \left(G_{TX;UL,UL} \times G_{RX;UL,UL}\right) \times \frac{1}{1 + P_{\text{noise}} / P_{RX;UL,UL}} \div S_U \tag{167}$$

so substituting the TPC formula (145) and combining and rearranging terms, gives

$$L_{UL,UL} \leq X \times \left(G_{TX;UL,UL} \times G_{RX;UL,UL}\right) \times \frac{1}{1 + P_{\text{noise}} / P_{RX;UL,UL}} \div S_U \tag{168}$$

In other words, the link budget limits the path loss and therefore the usable communication distance between the unlicensed devices. If the noise power is at least 10 dB below the power received from the licensed transmitter, then to first order we can ignore the noise term (we will return to this point in a moment). For a S_U value of 10 dB and the 95.8 dB value of X used above; and with half-wave dipoles for which the combined unlicensed device antenna gains are no greater than 4 dB; then the path loss between the unlicensed devices is limited to around 89.8 dB, which corresponds to a free-space distance of 58 meters at 12.72 GHz.

However, if we assume a 1 MHz signal bandwidth and a 10 dB noise figure (typical for low-cost consumer electronics), then we can calculate the total signal-to-impairment ratio for the link. Accounting for the noise power will reduce the free-space range between devices. This is demonstrated in Figure 41, which plots a distribution of achievable communication range for 50,000 random interferer locations for our running FS link example. With the additional consideration of noise, we see that communication ranges are limited to less than 40 meters, and that at 90% of locations the maximum range is less than 25 meters. These ranges will limit the utility of uses for such unlicensed devices, and furthermore this analysis is optimistic in the sense that the interference effect of only a single unlicensed transmitter was considered. A more realistic accounting for interference aggregation is the next topic to be discussed.

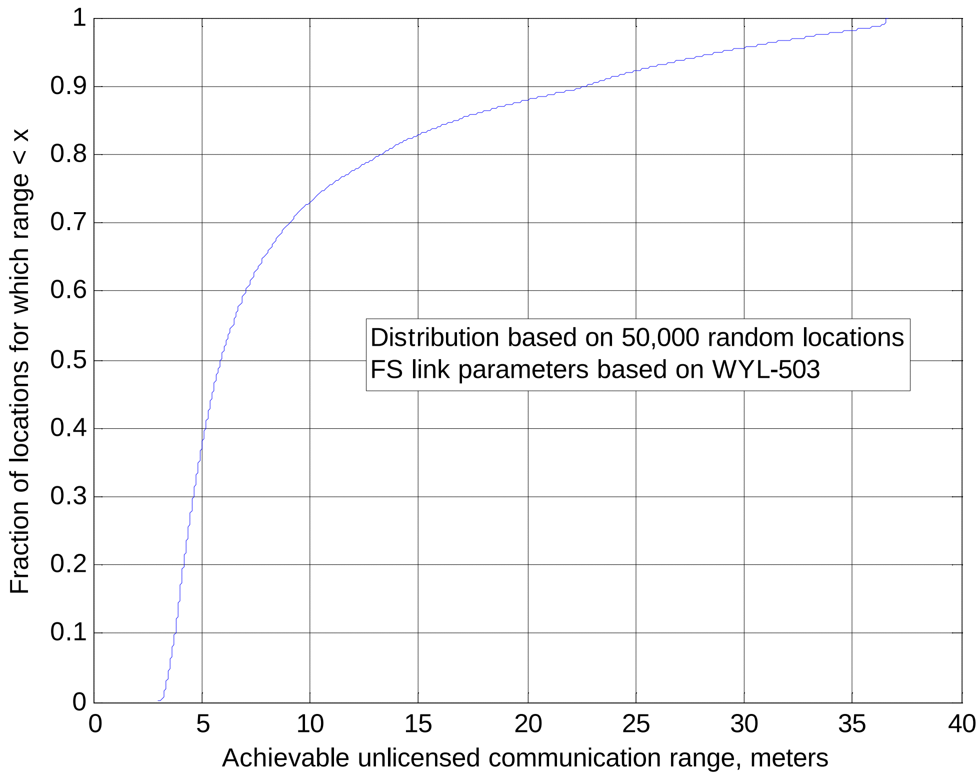

Figure 41 -- *Distribution of achievable unlicensed communication range for single-interferer analysis*

7.2.3 Interference Aggregation Effects

The previous comment about the utility of this spectrum-sharing method underscores another significant point that was not considered in the previous single-interferer analysis. Simply stated, the NPRM comment that victim receiver interference will be dominated by the single closest device is untrue on the surface. First, if the victim receiver antenna is mounted on a tower at thousands of wavelengths above ground level, then it is not possible for any single unlicensed interferer to be physically close to the victim. Indeed, there is no reasonable way to assert that a number of devices *cannot* be comparably close to the victim. Second, the interfering unlicensed transmitter will usually be part of a communications network employing more than a single transmitting device. However, we have already established that these underlaid networks can support communications only at relatively close range, so by default the cooperating transceivers will all offer comparable levels of interference to the victim receiver.

We can take advantage of our previous FS link example to get an idea of the amount of interference aggregation that might be obtained under real conditions. Note that evaluating distributions of the *X* parameter and allowable unlicensed transmit device power levels for 10,000 randomly located unlicensed devices provides a very conservative point for calculating aggregated interference. Distributing 10,000 devices over an area of 1,150 km^2 corresponds to an average area density of 8.7 devices per square kilometer, or about 115,000 square meters per device (equivalent to a circle of radius 191 meters).[16] We performed 100 Monte-Carlo trials of the random laydown of 10,000 unlicensed devices, and calculated the aggregated interference power as seen by the victim receiver, using the same TPC *X* parameter value of 95.8 dB. The distribution of results across the 100 trials is shown in Figure 42.

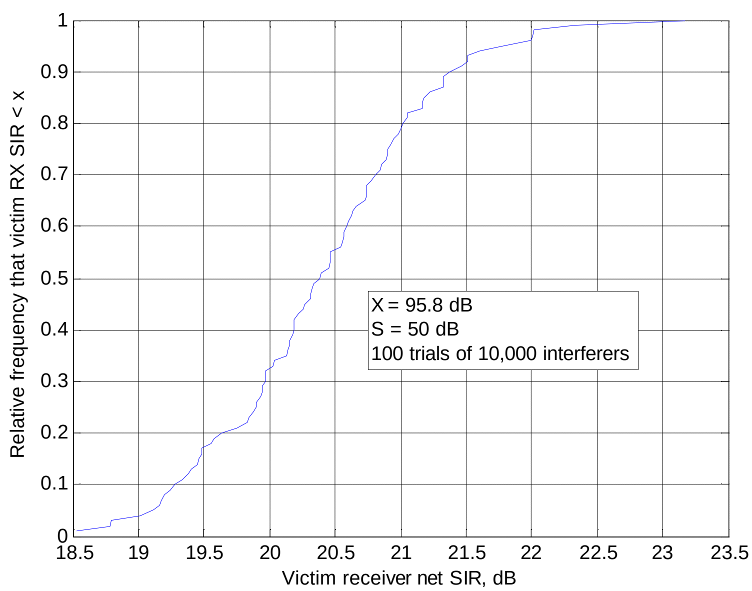

Figure 42 -- *Distribution of victim net SINR for 10,000 interferer placements*

First, we note that there is not a significant spread in the results. This is because we have used a large number of simulated interfering devices in this assessment.[17] However, we can establish with reasonable confidence that the effect of interference aggregation is to reduce the victim receiver's net SINR to a level well below that established with the

[16] This is significantly larger than the supportable communication range between devices as demonstrated in the previous section.

[17] A proper model for the temporal and spatial variability of the number of interferers would start with an average interferer density, which for a given study area results in an average number of interferers *K*. The actual number of interferers k_i transmitting in any given time interval *i* would be modeled as a Poisson point process, with parameter equal to *K*. Then the k_i interferers would be randomly placed assuming uniform spatial density over the study area; their individual transmit power levels determined; then their aggregated interference and corresponding impact on victim SINR calculated. Statistics would then be generated based on the expected variations over the time intervals.

parameter *S*. In this case, the degradation due to aggregation is on the order of 30 dB. Note also that this degradation is to first order invariant to the specific value of *S*, if the value of transmit power offset is adjusted accordingly with *S*. A lower victim receiver target SINR allows for proportionately higher transmit power levels for the unlicensed devices, since there is a decibel-for-decibel increase in the value of *X* with any decrease in *S*. Finally note that in practice, aggregation effects could be worsened if the interferers are more clustered in their distribution (this example uses a uniform distribution over area).

If we take into account the influence of aggregation, it will be necessary to further reduce the transmit power levels of the unlicensed devices, to approximately 30 dB below the already low levels seen in Figure 40. Without knowledge of its location and in particular of its ability to cause harmful interference to the victim receiver, it is not possible for any unlicensed device to behave otherwise.

7.2.4 Dynamic Frequency Selection Thresholds

In addition to transmit power control, the NPRM suggests that unlicensed devices wishing to transmit in spectrum licensed to fixed services should use dynamic frequency selection (DFS) as a means of ascertaining available channels. The NPRM goes on to suggest that values in the range of -64 to -62 dBm could be employed by the unlicensed device to determine the occupancy of the channel (¶44). The commission mentions that "these values generally are consistent with our proposals for DFS employed with UNII operation proposed in the 5470-5725 MHz band." In addition, there is consistency among

- the use of a DFS threshold at around -65 dBm;
- the use of a TPC rule allowing unlicensed devices to transmit at levels 70 dB to 90 dB above the power level received from the licensed FS transmitter;[18] and
- the perceived utility of unlicensed devices that have maximum transmit power levels in the range of +5 to +25 dBm (DFS threshold plus 70 dB to 90 dB).

However, the results obtained for our running example fixed link make it clear that such a DFS rule will not provide necessary protections to the incumbent FS link. In particular, consider Figure 40. The curve can also be interpreted as a distribution of the power levels received from the FS transmitter at random locations in the study area, once the value of *X* is subtracted from the abscissa values in accordance with the TPC rule. Fully 90% of the locations would not permit a transmit power level above -10 dBm. With *X* = 95.8 dB, this implies that power levels measured by the unlicensed receiver would be less than approximately -106 dBm – which coincidentally is about the level of the thermal noise floor at a 6 MHz bandwidth – at 90% of randomly selected locations. However, we have seen ample evidence that unlicensed device locations where low power levels are measured from the FS transmitter can still result in significant interference to the FS receiver from unlicensed device transmissions.

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[18] NOI/NPRM, at ¶43

To summarize these observations, DFS threshold values in the vicinity of -65 dBm are far too high to infer that a given frequency channel is not in use by a neighboring fixed point-to-point microwave link. In addition, it is not possible to assume that unlicensed device locations at which power is measured below such a DFS threshold cannot cause significant interference to the paired FS receiver.

A simple example can clearly illustrate this point. For our running example based on parameters for CARS license WYL-503, consider an unlicensed device located 100 meters from the base of the transmitter tower along the centerline of the link. Straightforward calculations show that at this location, the unlicensed device measures -78.5 dBm power from the licensed transmitter; if it transmits at a power level 90 dB higher than it receives, then it transmits at +11.5 dBm; at that transmit power level, the corresponding interference power seen by the licensed receiver is -81.7 dBm; and finally the desired signal into licensed receiver is -28.9 dBm. See Table 2 for details of these calculations.

Term	Licensed TX to Unlicensed RX	Unlicensed TX to Licensed RX	Licensed TX to Licensed RX
TX power into antenna, dBm	+18.0	+11.5	+18.0
Plus TX antenna gain, dB	0.1	2.1	47.6
Less path loss to RX antenna, dB	96.3	142.1	142.1
Plus RX antenna gain, dB	-0.3	46.6	47.6
Total RX power, dBm	**-78.5**	**-81.9**	**-28.9**

Table 2 -- *Interferer location example for DFS based on CARS licensed WYL-503*

For this single interferer location, where the measured power is on the order of 15 dB below the threshold proposed in the NPRM, the corresponding SIR at the licensed receiver is 53 dB; not high enough given the likelihood of significant interference aggregation, and the SINR engineering requirement on many CARS links of 50 dB.

7.3 Two-Way Links

Although less commonly employed than one-way links, a two-way FS link might seem to provide a promising opportunity for measurement-based spectrum sharing with unlicensed devices. Since the transmitter and receiver are essentially co-located, it seems possible that the unlicensed transceiver could better ascertain the susceptibility of the victim receiver to interference by making measurements of transmit power. However, there are two complications that reduce the utility in this approach. First, the unlicensed device would need to make measurements on two frequencies: the frequency of the licensed FS transmitter, to identify its proximity to the victim receiver; and the frequency at which it intends to transmit, to ensure that no other interference is present that would impair the operation of the unlicensed device. Second, and more seriously, the duplexed frequency often uses a cross-polarized mode of the antenna, so cross-polarization (XPOL) discrimination must be factored into the calculations in order for the unlicensed device to assess the true susceptibility of the victim receiver from the transmissions of the co-located FS transmitter.

For example, assume the unlicensed device uses a simple vertically-oriented half-wave dipole. If the licensed transmitter uses a horizontal polarization, the unlicensed device may measure very low received power on that frequency; assume that it is far from the transceiver; and calculate that it can use a fairly high transmit power level on the duplexed pair. However, the unlicensed device would be actually much closer to the licensed transceiver than the measurement suggests, and the licensed receiver would be very susceptible to interference since it is using the same vertical polarization as the interferer.

These complications, combined with the relatively low number of two-way links compared to one-way links, make further consideration of this topic unwarranted.

7.4 Feedback-based ITemp Management

The NOI/NPRM make scant mention of feedback-based ITemp approaches to sharing spectrum with FS point-to-point links, particularly in the detailed NPRM discussion. Feedback methods would be far more reliable than open-loop methods, assuming the feedback is based on the victim receiver's view of the interference environment. However, for FS point-to-point links, the victim's view of interference is intimately tied to the characteristics of the receive antenna. It would be impractical to co-locate a separate monitoring system antenna with similar characteristics as the victim receiver's antenna; for example, an eight-foot parabolic reflector is large, expensive, and difficult to mount on a tower several tens of meters or higher above ground level. Sharing of the victim receiver's antenna between the FS link and a monitoring system is also complicated because all components of the link, including splitters, circulators and cabling, factor into link budgets; a "passive tap" for the monitoring system introduces further loss that must be compensated elsewhere in the link design.

7.5 Unlicensed Sharing without Measurements: Exclusion Zone Assessment

The analyses to this point have identified some practical obstacles to reliable use of measurement-based methods that might facilitate the sharing of FS licensed spectrum with underlaid, low-power unlicensed devices. An alternative is to consider the use of unlicensed devices that employ GPS technology, together with databases of FS transceiver locations, to define physical exclusion zones around victim receivers. An obvious solution would be to allow unlicensed devices to use licensed FS frequencies as long as they were beyond the radio horizon of any potential co-channel victim FS receiver. A more aggressive approach would be to define an exclusion zone out to some distance from the potential victim, within which no unlicensed devices could be used co-channel to the licensed FS link; and to allow unlicensed use outside that exclusion zone. This idea is illustrated in Figure 43. We now consider the interference impact of co-channel unlicensed devices within the shaded interference region upon the FS receiver.

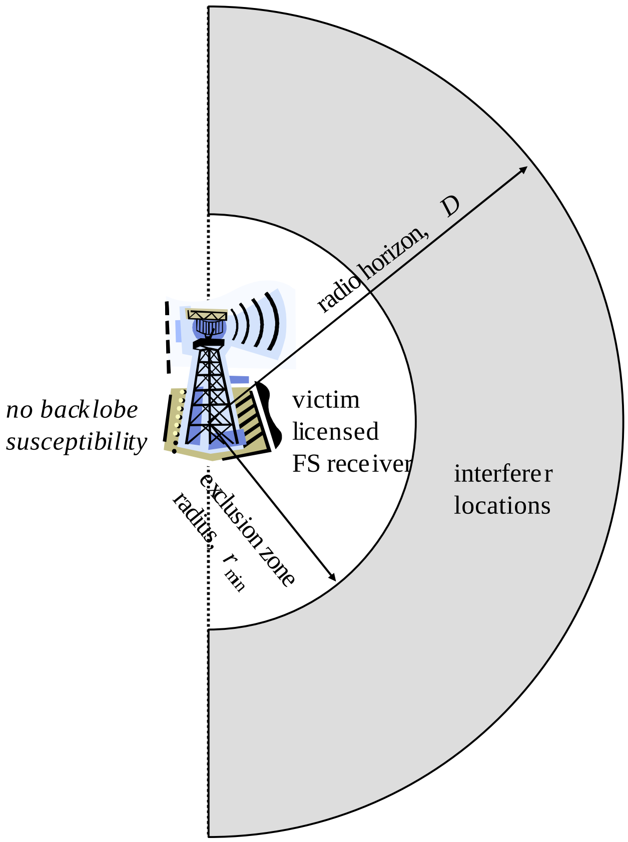


Figure 43 -- *Exclusion zone concept for FS link*

Recall that Section 3 contains an analysis of the statistics of interference from a single interferer located outside of an exclusion zone of radius $r_{\min}$. If the interferer is located at a radius d_u, then the interference it presents to the victim receiver is $I(d_u) = I_{\max}(d_u / r_{\min})^{-g}$, where $I_{\max} = ar_{\min}^{-g}$ is the interference that would be received from an unlicensed transmitter with $d_u = r_{\min}$. Also recall that for a uniformly randomly located interferer outside the exclusions zone, the probability density function of its distance from the victim receiver d_u is:

$$f_{d_u}(x) = \frac{2x}{D^2 - r_{\min}^2} \qquad r_{\min} \le x \le D \tag{169}$$

The number of interferers in the zone of interest is properly modeled as a Poisson density. Given an average interferer area density r_u, and assuming that all interferers transmit at a common constant power level, then the number of interferers contributing to the total interference power seen by the victim receiver is k; and the Poisson parameter K is given by the average number of such interferers, $pr_u\left(D^2 - r_{\min}^2\right)$. Taking this into account, then the total interference seen by the victim receiver is

$$I_{TOT} = \sum_{i=1}^{k} I_{\max}\left(d_u(i)/r_{\min}\right)^{-g} \tag{170}$$

where $d_u(i)$ is the distance of the i^{th} interferer from the victim. Following the earlier aggregation analysis, we wish to calculate mean and variance of I_{TOT} for a given value of D and $r_{\min}$. To do this it is necessary to calculate expectations of I_{TOT} and I_{TOT}^2 over both the distribution of the number of interferers k, and the distribution of their distances $d_u(i)$ from the victim receiver. Fortunately these two distributions are independent, so we can first calculate these statistics over the distance from the victim receiver d_u and then over the number of interferers k:

$$\overline{I_{TOT}} = \mathop{E}_{k}\left[\sum_{i=1}^{k} \mathop{E}_{d_u}\left[I_{\max}\left(d_u(i)/r_{\min}\right)^{-g}\right]\right] \tag{171}$$

and

$$\overline{I_{TOT}^2} = \mathop{E}_{k}\left[\mathop{E}_{d_u}\left[\left(\sum_{i=1}^{k} I_{\max}\left(d_u(i)/r_{\min}\right)^{-g}\right)^2\right]\right] \tag{172}$$

Building on the previous analysis of section 3, we arrive at the following expressions for the case of free-space path loss ($g = 2$), with the substitution of $a / r_{\min}^2$ for $I_{\max}$:

$$\overline{I_{TOT}} = 2par_u \ln\left(\frac{D}{r_{\min}}\right) \tag{173}$$

$$\overline{I_{TOT}^2} = a^2 r_u^2 p^2 \left(D^2 - r_{\min}^2\right)\left[\frac{1}{r_{\min}^2} - \frac{1}{D^2} + 4pr_u \ln\frac{D}{r_{\min}}\left(1 + pr_u\left(D^2 - r_{\min}^2\right)\ln\frac{D}{r_{\min}}\right)\right] \tag{174}$$

and

$$s_{\bar{I}}^2 = \overline{I_{TOT}^2} - \left(\overline{I_{TOT}}\right)^2 = a^2 pr_u\left(\frac{1}{r_{\min}^2} - \frac{1}{D^2}\right) \tag{175}$$

Finally, note that the ratio of the standard deviation of the interference to its mean is

$$\frac{s_{\bar{I}}}{\overline{I}_{TOT}} = \frac{1}{2Dr_{\min}\ln(\frac{D}{r_{\min}})}\sqrt{\frac{D^2 - r_{\min}^2}{p r_u}} \tag{176}$$

These results are intuitively sensible for free-space (square-law) propagation. First, the mean interference grows without bound if *D* becomes arbitrarily large. Second, the normalized standard deviation of the interference tends to zero as *D* becomes arbitrarily large; in other words, as *D* gets very large, the number of interferers in the shaded region becomes very large, and the mean of the total interference grows to dominate any variation in the total interference.

The purpose of the exclusion zone is to limit the impact of interferers upon the victim receiver. This can be quantified by establishing an inequality for victim receiver's average SIR, after some algebraic manipulations and using terminology from earlier in this chapter:

$$\frac{P_{RX;L,L}}{\overline{I}_{TOT}} = \frac{P_{TX,L}G_{TX,L}}{2pP_{TX,UL}G_{TX,UL}\,r_u D_{L,L}^2 \ln\left(\frac{D}{r_{\min}}\right)} \geq S \tag{177}$$

Note that at the moment there is no accounting for the antenna gain of the victim receiver, as the analysis assumes that it is isotropic. We will return to this point.

A rearrangement of (177) illustrates several tradeoffs that can be explored in the process of protecting the victim receiver:

$$2p \times r_u \times S \times \ln\left(\frac{D}{r_{\min}}\right) \leq \frac{P_{TX,L}G_{TX,L}}{P_{TX,UL}G_{TX,UL}} \times \frac{1}{D_{L,L}^2} \tag{178}$$

The right hand side of (178) is the EIRP ratio between the licensed transmitter and any single unlicensed transmitter. The left-hand side has several quantities that can be traded; for example:

- for a given victim threshold SIR value *S*, increasing the interferer density r_u requires an increase in the exclusion zone radius $r_{\min}$;
- for a given interferer density, increasing the unlicensed transmitter EIRP $P_{TX,UL}G_{TX,UL}$ requires an increase in the exclusion zone radius $r_{\min}$;
- a reduction in victim threshold SIR value *S* can afford a proportional increase in interferer density r_u, or a decrease in exclusion zone radius $r_{\min}$, or an increase in unlicensed transmitter EIRP $P_{TX,UL}G_{TX,UL}$.

Note that for standard FS link budgets that assume free-space propagation along the link path, the licensed transmitter EIRP $P_{TX,L}G_{TX,L}$ will vary proportionally with the square of the path length $D_{L,L}$, so the ratio of $P_{TX,L}G_{TX,L}$ to $D_{L,L}^2$ will be constant.

How might this analysis shed light on the total interference that would be seen by a victim FS point-to-point link receiver?

1. Define D to be the distance to the radio horizon given the receiver antenna height; interferers beyond the radio horizon would arguably not contribute to the victim's total interference under usual propagation conditions;
2. Simplify the effect of the elevated, directional receiver antenna by accounting for on-boresight gain for the desired signal in the victim receiver's SIR calculation.

For an antenna height h in feet that is small relative to the radius of the earth, the distance D to the radio horizon in miles is well-approximated by [3]

$$D = \sqrt{\frac{3bh}{2}} \tag{179}$$

where b is the ratio of the effective (refraction-adjusted) to the true radius of the earth; we will use the usual value of 4/3.

Using parameter values from our running point-to-point link example, an example of a density versus exclusion tradeoff for fixed values of the other parameters is shown in Figure 44. Note that the plot is normalized to the value of the supported density at $r_{\min} = D/2$, i.e. the normalized density is defined to be unity when the exclusion zone radius extends halfway to the radio horizon.[19] We see that the supported density increases rapidly as $r_{\min}$ approaches D, because the area of the shaded interference region of Figure 43 decreases as $D^2 - r_{\min}^2$; for example, as $r_{\min}$ goes from 0.9D to 0.95D, the interference region area decreases by a factor of 1.95, and not surprisingly the supportable interferer density roughly doubles at the same time.

[19] Consideration of the absolute values of supportable interferer density is not reasonable at this point, because the analysis starts from the assumption that the victim receiver's antenna is isotropic. However, the shape of the density versus exclusion zone radius curve is worth observing.

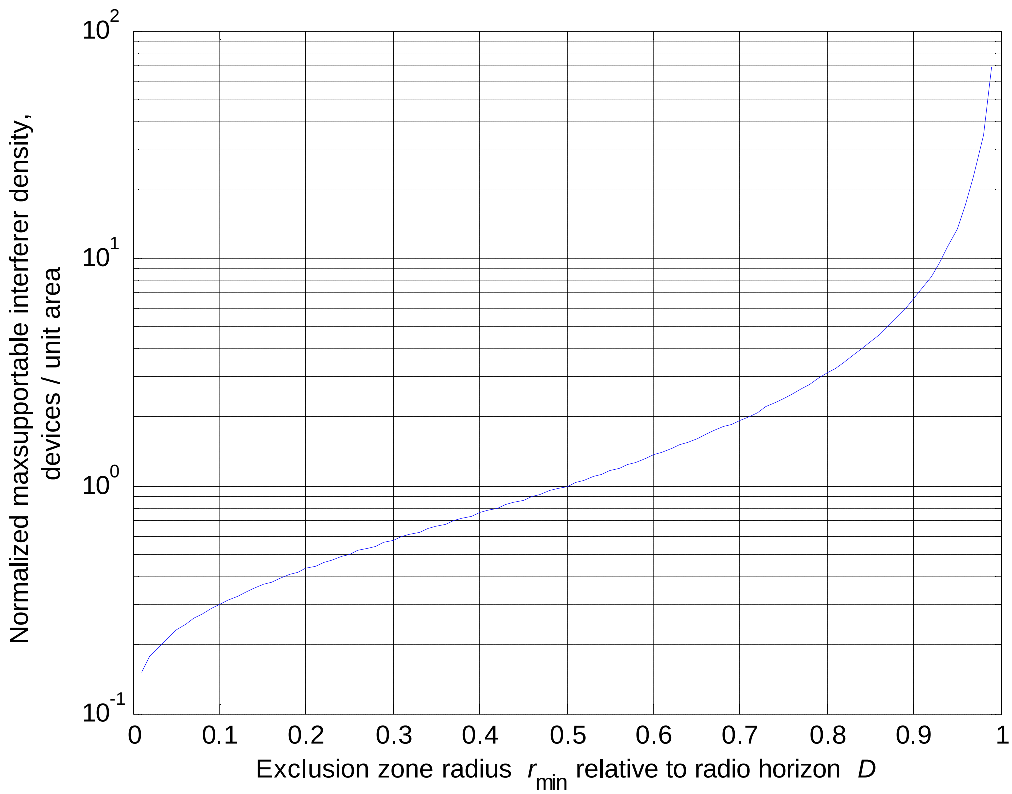


Figure 44 -- *Example density vs. exclusion zone tradeoff analysis*

Now that we have some idea of the relative relationship between exclusion zone radius and supported interferer density, let's examine the interference effects of absolute densities of underlaid unlicensed devices, using more realistic modeling of the victim receiver antenna (discrimination and elevation).

A simulation was developed that determines the net victim receiver SIR for the running FS link example as a function of exclusion zone radius, given the interferer density in transmitters per unit area and the common transmit power level used by those interferers. For a given value of the exclusion zone radius, the interferer density was used to calculate the mean number of transmitters in the area, which were then randomly distributed from a uniform spatial density within the interference region. The interference power into the victim FS receiver was calculated for each unlicensed transmitter using path loss and antenna gain factors, and then the interference powers for all unlicensed transmitters were summed to provide a total interference number. This power sum was divided into the signal received from the licensed transmitter, to establish the victim's SIR for that case. Results for three specific cases of interferer density are plotted in Figure 45. Note that the radio horizon in this instance is at a radius of approximately 34.5 km from the victim receiver.

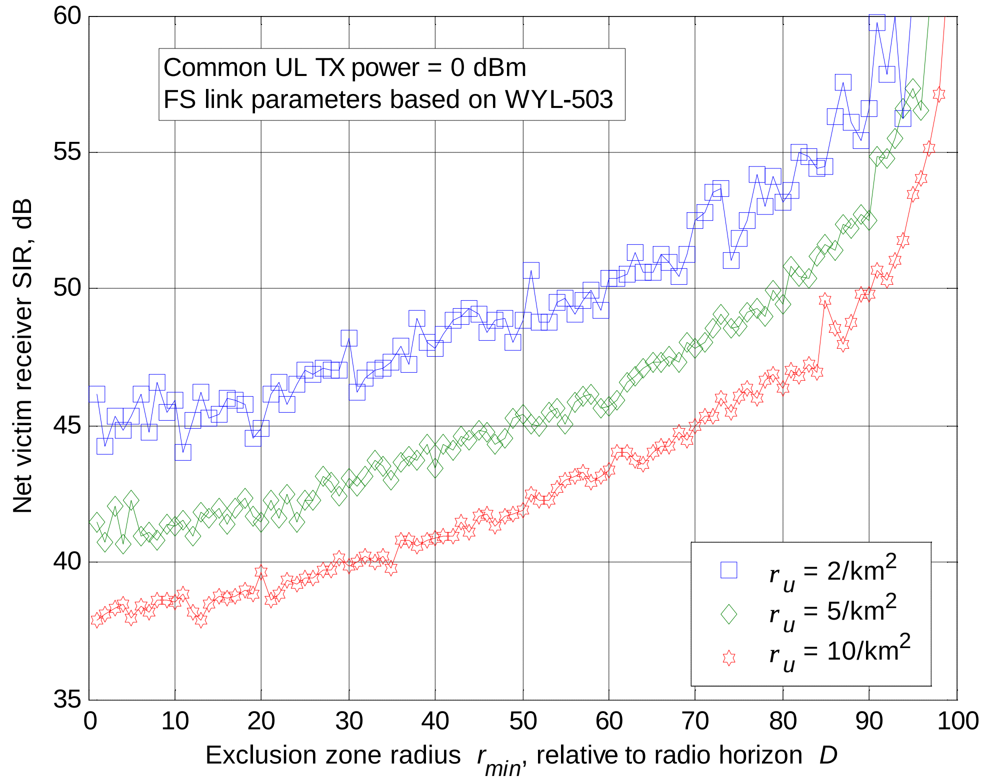


Figure 45 -- *Victim receiver SIR for exclusion zone simulations*

These curves are somewhat "noisy" because only a single random laydown of interferers was performed for each value of exclusion zone radius. Additional Monte-Carlo trials, with averages across many random laydowns of the unlicensed interferers, and perhaps across many random numbers of unlicensed interferers (using Poisson-distributed trials instead of the mean number), will smooth out the fine-scale fluctuations. However, the general behavior of the victim receiver net SIR versus the exclusion zone radius is clearly displayed.

As would be expected, doubling the interferer density for any exclusion zone radius decreases the victim's net SIR by approximately 3 dB (i.e., the curve for $r_u = 10 / \text{km}^2$ is approximately 3 dB below the curve for $r_u = 5 / \text{km}^2$). Note also that to maintain the victim's SIR, increasing the interferer density requires increasing the exclusion zone radius. The amount of increase in the exclusion zone radius is approximated by the increase in radius that maintains a constant average number of interferers in the area of interest. In fact this increase is an upper bound, because those interferers in the smaller interference region experience greater path loss to the victim and therefore offer less interference than would be seen by the same total number of interferers inside a larger interference region (e.g. outside a smaller exclusion zone). It can be shown that if the exclusion zone radius is parameterized as some fraction d of the distance to the radio horizon, then to decrease the area by a factor of h ($h > 1$), the exclusion zone radius must be increased to

$$d' = \sqrt{\frac{d^2}{h} + \left(1 - \frac{1}{h}\right)} \tag{180}$$

For illustration, at a victim net SIR of 50 dB, Figure 45 shows an exclusion zone radius with $r_{\min} = 0.60D$ for $r_u = 2$ / km^2, $r_{\min} = 0.81D$ for $r_u = 5$ / km^2, and $r_{\min} = 0.91D$ for $r_u = 10$ / km^2. Given the $r_u = 2$ / km^2 result, the formula provides the upper bounds as $r_{\min} = 0.86D$ for $r_u = 5$ / km^2 and $r_{\min} = 0.93D$ for $r_u = 10$ / km^2.

Finally, note that the simulated net victim SIR is reduced decibel for decibel with any increase in the common unlicensed device TX power that is chosen. Again, in the results of Figure 45, we can achieve a victim SIR of 50 dB for $r_u = 2.0$ per km^2 and unlicensed transmit power of 0 dBm with $r_{\min} = 0.60D$; or a victim SIR of approximately 43 dB for $r_u = 10.0$ per km^2 with the same size exclusion zone and 0 dBm unlicensed transmit power; or a victim SIR of 50 dB with $r_u = 10.0$ per km^2, the same size exclusion zone, and -7 dBm transmit power.

The exclusion zone analysis and simulation results assume that the unlicensed devices are uniformly spatially distributed in the area of interest. This is an analytically tractable but unrealistic simplification; low-power unlicensed transmitters will probably be used for short-range communications, and therefore will be distributed in networked “clumps” of varying sizes over the interference area. It is not apparent that a nonuniform clumped distribution provides demonstrably higher or lower interference than the uniform distribution assumption.

For a given victim receiver SIR threshold, the predicted supportable densities must include reasonable engineering margin above device densities which might reasonably be expected to occur. If this cannot be ensured; or if the devices might have difficulty ascertaining the exclusion region with a reasonable degree of accuracy; or if the victim receiver’s antenna has some susceptibility to backlobe interference (which was ignored in the analysis); then the conservative rule would be to define the exclusion region out to the radio horizon and practically eliminate the interference effects of the unlicensed devices.

7.6 Summary

In this section we have examined the potential impact of spectrum sharing between presumably low-power unlicensed communication devices and licensed fixed service point-to-point links at microwave frequencies. Particular attention was focused on the simplified interference temperature methods identified by the FCC in the interference temperature NPRM. Characteristics of realistic FS links were incorporated into analytical and simulation models to quantify the impact of unlicensed interferers upon the performance of the FS receiver. Two decision policies for the unlicensed device to transmit on a licensed FS frequency – the use of a power threshold in a dynamic frequency selection procedure, and the use of a transmit power control scheme based on received signal power levels – were evaluated. An alternative decision policy, based not on measurements but instead on geolocation of the unlicensed device relative to the known location of the potential victim receivers, was also examined.

The major findings of this section are summarized in the following list:

1. Licensed FS links are engineered for high availability and reliability. Significant power margin is placed into the link budget to account for the effects of time-variant impairments such as multipath or rain fading, on top of other impairments such as interference and noise. The introduction of additional interference from underlaid unlicensed devices and systems that frequencies with FS links will reduce the design margin of those links. It is the nature of the time-varying impairments that reduced design margins will result in increased service outage, with corresponding costs – in terms of the diminished value of a less-reliable service, or the cost of engineering mitigations to restore the intended reliability – to the link operator or service provider.
2. The NPRM proposes an open-loop system for spectrum sharing between FS links and unlicensed devices in the interference temperature NPRM. In such a system, where the unlicensed device would use measurements of the licensed transmitter to decide its transmit power and then employ conservative rules to avoid interfering with the incumbent service, there would be little remaining utility (e.g. communication range between unlicensed devices) for the unlicensed devices to exploit. Moreover, this approach includes no way to account for interference aggregation effects.
3. The use of a DFS threshold for the measurement of transmitter power is not a reliable way to determine if a licensed link is in use, and in particular whether or not the unlicensed device can use the frequency without causing harmful interference to the licensed service.
4. Non-measurement methods – such as requiring unlicensed devices to use GPS and consult a database of licensed receiver locations – may be more reliable ways to allow unlicensed devices to use frequency bands that are licensed to microwave fixed services, without interfering with licensed operations. Because unlicensed device density is difficult to foresee, such a location-based approach should require that unlicensed devices be beyond the radio horizon of the licensed receiver in order to operate.

On the basis of the results of this section, it would be premature to proceed with the proposals outlined in the portions of the interference temperature NPRM that pertain to FS spectrum.

Section 7 References

[1] Bell Telephone Laboratories, *Transmission Systems for Communications*, Fifth Edition, 1982. See Chapter 8, “Objectives”, pp. 161-165.

[2] L. J. Greenstein and M. Shafi, “Outage calculation methods for microwave digital radio,” *IEEE Communications Magazine*, Volume 25, Issue: 2, February 1987, pp. 30-39.

[3] *Reference Data for Radio Engineers*, Sixth Edition, Howard W. Sams & Co., Indianapolis, IN, 1981.

8. Annex A: CDF of the Aggregate Interference

This Annex supports the analysis in Section 3, and derives the cumulative distribution function (CDF) of the total interference power as seen by a receiver surrounded by randomly-located interfering transmitters with a given average spatial density (*e.g.*, devices per square km).

The Model

Assume a normalized distance scale such that the average density of interference sources transmitting within the band of interest at a given time is $1/p$ (interferers per normalized unit area). If the "victim" receiver is at the center of a circle of a normalized radius $\sqrt{K}$, the expected (average) number of interference sources within the circle is *K*. Assuming that interfering transmitters are randomly-distributed over area in a uniform fashion, the *actual* number of active interfering transmitters within the circle at a given time can be modeled as a Poisson-distributed random variable **J** with discrete probability density function (pdf):

$$P_{\mathbf{J}}(k) = \Pr\{\mathbf{J} = k\} = \frac{e^{-K} K^k}{k!} \qquad (1)$$

where the notation $\Pr\{\cdot\}$ represents the probability of the indicated event. The normalized power received at the base station from the k^{th} interfering transmitter a normalized distance s_k away from it is $z_k = s_k^{-g}$. The total power received from interfering transmitters within the circle of normalized radius $\sqrt{K}$ is:

$$Z_K = \sum_{k=1}^{J} z_k \ . \qquad (2)$$

With interferers that are randomly distributed over area, the pdf of s_k is:

$$f_{s_k}(s) = \frac{2s}{K}, 0 \le s \le \sqrt{K}. \qquad (3)$$

Hence, the pdf of z_j is:

$$f_{z_j}(z) = \frac{2}{gK} z^{-(g+2)/g}, \quad K^{-g/2} \le z \le \infty \qquad (4)$$

The Characteristic Function of the Aggregate Interference

The characteristic function of Z_K is:

$$\Phi_{Z_K}(w) = E\left[e^{jwZ_K}\right] = \int_0^{\infty} f_{Z_K}(z) e^{jwz} dz \,, \tag{5}$$

which is the Fourier transform of $f_{Z_K}(z)$. The lower limit is 0 rather than $-\infty$ in this case because Z_K represents power and therefore is non-negative.

Assuming the $\{z_k\}$ are independent and identically-distributed (i.i.d.), (2) and (5) yield:

$$\Phi_{Z_K}(w)\big| J = E\left[\exp\left(jw\sum_{k=1}^{J} z_k\right)\right] = \left(E\left[e^{jwz_k}\right]\right)^J \,. \tag{6}$$

Taking the expectation over *J* using (1) gives:

$$\Phi_{Z_K}(w) = \sum_{n=0}^{\infty} \frac{e^{-K}K^n}{n!}\left[\Phi_{Z_k}(w)\right]^n = \exp\left\{K\left[\Phi_{z_k}(w) - 1\right]\right\}. \tag{7}$$

Thus, Z_K has a compound Poisson distribution [1]. Letting $n = 2/g$, (4) gives the characteristic function of z_k as:

$$\Phi_{z_k}(w) = \int_0^{\infty} f_{z_k}(z) e^{jwz} dz = \frac{2}{gK} \int_{K^{-g/2}}^{\infty} z^{-1-n} e^{jwz} dz \,. \tag{8}$$

The "second characteristic function" of Z_K is defined as the natural logarithm of the characteristic function [2]. Hence,

$$\Psi_{Z_K}(w) = \ln \Phi_{\mathbf{Z}_K}(w) = K\left[\Phi_{\mathbf{z}_k}(w) - 1\right] = \left(n \int_{K^{-1/n}}^{\infty} z^{-1-n} e^{jwz} dz\right) - K \,. \tag{9}$$

Integrating by parts and recalling that $\mathbf{Z} = \lim_{K\to\infty} \mathbf{Z}_K$ gives:[20]

$$\begin{aligned} \Psi_{\mathbf{Z}}(w) &= \lim_{K\to\infty} \Psi_{\mathbf{Z}_K}(w) = jw \int_0^{\infty} z^{-n} e^{jwz} dz \\ &= \begin{cases} -|w|^n \,\Gamma(1-n) e^{-jpn/2}, & w \ge 0 \\ -|w|^n \,\Gamma(1-n) e^{jpn/2}, & w < 0 \end{cases} \end{aligned} \tag{10}$$

[20] See also [3], p. 10, §1.3, #1, and p. 68, §2.3, #1.

where $\Gamma(\cdot)$ is the Gamma function [4].

The PDF and CDF of the Aggregate Interference

The pdf of **Z** is given by the Fourier inversion formula:

$$
\begin{aligned}
f_{\mathbf{Z}}(z) &= \frac{1}{2p}\int_{-\infty}^{\infty}\Phi_{\mathbf{Z}}(w)e^{-jwz}dw = \frac{1}{2p}\int_{-\infty}^{\infty}e^{\Psi_{\mathbf{Z}}(w)}e^{-jwz}dw \\
&= \frac{1}{2p}\sum_{k=0}^{\infty}\int_{-\infty}^{\infty}\frac{\left[\Psi_{\mathbf{Z}_K}(w)\right]^k}{k!}e^{-jwz}dw\,.
\end{aligned}
\tag{11}
$$

Letting $x = -\Gamma(1-n)e^{-jpn/2} = \Gamma(1-n)e^{jp(1-n/2)}$, $\Psi_{\mathbf{Z}}(w) = w^n x$ for $w \geq 0$, and $\Psi_{\mathbf{Z}}(w) = -w^n x^*$ for $w < 0$ (where x^* denotes the complex conjugate of *x*), and (11) becomes:

$$
f_{\mathbf{Z}}(z) = \frac{1}{2p}\sum_{k=0}^{\infty}\frac{1}{k!}\int_{0}^{\infty}\left[\left(w^n x\right)^k e^{-jwz} + \left(w^n x^*\right)^k e^{jwz}\right]dw \tag{12}
$$

The integrals $\int_0^\infty w^{kn}e^{-jwz}dw$ and $\int_0^\infty w^{kn}e^{jwz}dw$ can be evaluated using a form of Euler's integral for the Gamma function ([4], p. 255):

$$
\Gamma(y) = x^y\int_0^\infty w^{y-1}e^{-wx}dw\,, \quad \mathrm{Re}\,y > 0\,, \quad \mathrm{Re}\,x > 0\,, \tag{13}
$$

where $\mathrm{Re}\{\cdot\}$ denotes the real part of the complex argument and the condition on x is necessary to assure convergence of the integral.

Letting $x = z - je$, where *z* and *e* are real and positive, (13) gives:

$$
\begin{aligned}
\int_0^\infty w^{kn}e^{-jwz}dw &= \lim_{e\to 0}\int_0^\infty w^{kn}e^{-jwx}dw = \lim_{e\to 0}\frac{\Gamma(kn+1)}{(jx)^{kn+1}} \\
&= \frac{\Gamma(kn+1)}{(jz)^{kn+1}} = \frac{\Gamma(kn+1)}{z^{kn+1}e^{-jp(kn+1)/2}}\,, \quad z > 0
\end{aligned}
\tag{14a}
$$

and

$$\int_0^\infty w^{kn} e^{jwz} dw = \lim_{e \to 0} \int_0^\infty w^{kn} e^{-jwx^*} dw = \lim_{e \to 0} \frac{\Gamma(kn+1)}{(jx^*)^{kn+1}} \\ = \frac{\Gamma(kn+1)}{(-jz)^{kn+1}} = \frac{\Gamma(kn+1)}{z^{kn+1} e^{jp(kn+1)/2}}, \quad z > 0 \quad . \tag{14b}$$

Eq. (12) then becomes:

$$f_{\mathbf{z}}(z) = \frac{1}{2p} \sum_{k=0}^{\infty} \frac{\Gamma(kn+1)}{k!\, z^{kn+1}} \left[x^k e^{-jp(kn+1)/2} + (x^*)^k e^{jp(kn+1)/2} \right] \\ = \frac{1}{p} \sum_{k=0}^{\infty} \frac{\Gamma(kn+1)[\Gamma(1-n)]^k}{k!\, z^{kn+1}} \cos[kp(1-n/2) - p(kn+1)/2] \\ = \frac{1}{p} \sum_{k=0}^{\infty} \frac{\Gamma(kn+1)[\Gamma(1-n)]^k}{k!\, z^{kn+1}} \sin kp(1-n), \quad z > 0. \tag{15}$$

The argument of the sum vanishes for $k = 0$ and (15) can be written as:[21]

$$f_{\mathbf{z}}(z) = \frac{1}{p\, z} \sum_{k=1}^{\infty} \frac{\Gamma(kn+1)}{k!} \left[\frac{\Gamma(1-n)}{z^n} \right]^k \sin kp(1-n), \qquad z > 0. \tag{16}$$

The CDF is then:

$$F_{\mathbf{z}}(z) = 1 - \int_z^\infty f_{\mathbf{z}}(x) dx = 1 - \frac{1}{p} \sum_{k=1}^{\infty} \frac{\Gamma(kn)}{k!} \left[\frac{\Gamma(1-n)}{z^n} \right]^k \sin kp(1-n), \quad z > 0. \tag{17}$$

For $z >> 1$, the first term in the series dominates. Since $\Gamma(n)\Gamma(1-n) = p \csc(1-n)$,[22] $F_{\mathbf{z}}(z) \cong 1 - z^{-\nu}$ for $z >> 1$.

Closed-Form Expressions for Fourth-Power Propagation

For the special case of $g = 4$ $(n = 1/2)$, (16) and (17) can be reduced to closed form.[23] Since $\sin kp/2$ vanishes for even values of *k*, and $\Gamma(1/2) = \sqrt{p}$, (16) becomes:

$$f_{\mathbf{z}}(z) = \frac{1}{pz^{3/2}} \sum_{k=0}^{\infty} \frac{\Gamma(k+3/2) p^{k+1/2}}{(2k+1)!\, z^k} (-1)^k, \quad g = 4, \quad z > 0. \tag{18}$$

---

[21] Expressions equivalent to (16) and (17) are given in [5]. However, the expression given in that paper for the CDF is incorrect and actually represents the complementary distribution.

[22] [4], p. 256, 6.1.17.

[23] This also is noted in [5].

With the identity:[24]

$$\Gamma[2(k+1)] = \frac{\Gamma(k+1)\Gamma(k+3/2)\,2^{2k+3/2}}{\sqrt{2p}} \qquad (19)$$

and the fact that $\Gamma[2(k+1)] = \Gamma(2k+2) = (2k+1)!$, (19) yields:

$$\frac{\Gamma(k+3/2)}{(2k+1)!} = \frac{\sqrt{2p}}{k!\,2^{2k+3/2}} = \frac{\sqrt{p}}{2\cdot k!\cdot 4^k}, \qquad (20)$$

and (18) is seen to be:

$$f_Z(z) = \frac{1}{2z^{-3/2}} \sum_{k=0}^{\infty} \frac{(-p/4z)^k}{k!} = \frac{1}{2} z^{-3/2} e^{-p/4z}, \quad g = 4,\ z > 0\,. \qquad (21)$$

In a similar manner, (17) reduces for $g = 4$ to:

$$F_Z(z) = 1 - \frac{1}{p} \sum_{k=0}^{\infty} \frac{\Gamma(k+1/2)}{(2k+1)!} \frac{p^{\,k+1/2}}{z^{k+1/2}} (-1)^k, \quad g = 4,\ z > 0. \qquad (22)$$

With the identity:[25]

$$\Gamma(2k) = \frac{\Gamma(k)\Gamma(k+1/2)2^{2k-1/2}}{\sqrt{2p}} \qquad (23)$$

and with $(2k+1)! = (2k-1)!\cdot(2k+1)\cdot 2k$ and $\Gamma(2k) = (2k-1)!$,

$$\frac{\Gamma(k+1/2)}{(2k+1)!} = \frac{2\sqrt{p}}{k!(2k+1)2^{2k+1}}\ . \qquad (24)$$

Substituting (24) into (22) yields:

---

[24] See [4], p. 256, 6.1.18.
[25] *Id.*

$$
\begin{aligned}
F_{\mathbf{Z}}(z) &= 1 - \frac{2}{\sqrt{p}} \sum_{k=0}^{\infty} \frac{p^{\,k+1/2}}{k!(2k+1) z^{k+1/2}} (-1)^k \\
&= 1 - \frac{2}{\sqrt{p}} \sum_{k=0}^{\infty} \frac{\left(\sqrt{p} / 2\sqrt{z}\right)^{2k+1}}{k!(2k+1)} (-1)^k \\
&= \operatorname{erfc}\left(\frac{\sqrt{p}}{2\sqrt{z}} \right), \quad g = 4
\end{aligned} \tag{25}
$$

where $\operatorname{erfc}(\cdot)$ is the complementary error function, defined as:[26]

$$
\operatorname{erfc}(x) = \frac{2}{\sqrt{p}} \int_x^{\infty} e^{-x^2} dx = 1 - \operatorname{erf}(x) \tag{26a}
$$

and $\operatorname{erf}(\cdot)$ is the error function

$$
\operatorname{erf}(x) = \frac{2}{\sqrt{p}} \int_0^{x} e^{-x^2} dx = \frac{2}{\sqrt{p}} \sum_{k=0}^{\infty} \frac{x^{2k+1}(-1)^k}{k!(2k+1)} \ . \tag{26b}
$$

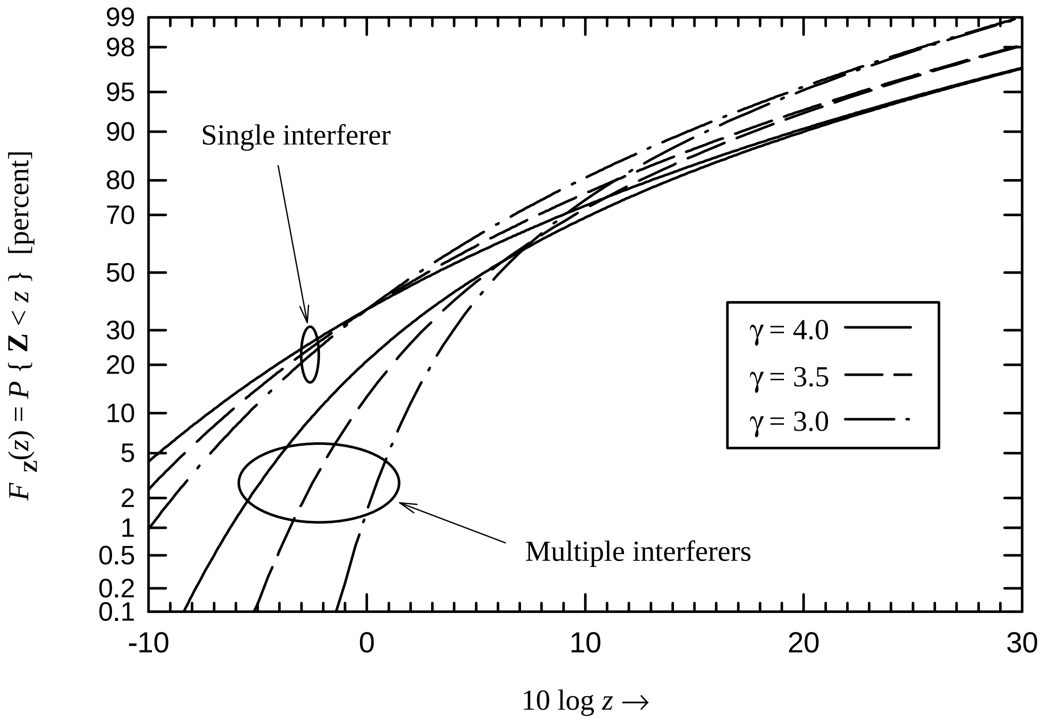
The Single-Interferer Case

In the context of this model, the CDF for the "single-interferer" case is easily derived by recalling that the average interference source density is $1/p$ active transmitters per unit area, and the normalized interference power from a source a distance *s* from the receiver is s^{-g}. Since the number of active transmitters within (normalized) distance *s* of the receiver is a Poisson-distributed random variable with mean value s^2, the probability that there are *no* active transmitters within that distance of the receiver is e^{-s^2}. Thus, since the normalized interference from a single source at a distance *s* is $Z = s^{-g}$, the probability $\Pr\{\mathbf{Z} < z\}$ for the "single-interferer" case is equal to the probability that there are no interfering transmitters within distance $s = z^{-1/g}$ of the receiver. Hence, for the single-interferer case,

$$
F_{\mathbf{Z}}(z) = \exp\left(-z^{-2/g}\right), \quad z \geq 0 \tag{27}
$$

The figure below shows $F_{\mathbf{Z}}(z)$ for $g = 3.0$, 3.5, and 4.0, for both the multiple-interferer and single-interferer cases.

[26] See [4], chapter 7.



Annex A References

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9. Annex B: Interference Correction Factors

The normal benchmark for the effect of interference is Gaussian noise; that is, for system-level analysis, it is often assumed that interference has the same effect on the victim device as Gaussian noise of the same average power level. In many cases, this is approximately true, once adjustment has been made for any difference in the bandwidths of the interfering signal and the passband of the victim receiver. That is, if the interfering signal has a bandwidth of W_I and the victim receiver has a bandwidth of W_V, then the bandwidth correction factor is:

$$BCF = \begin{cases} \dfrac{W_V}{W_I} & W_V < W_I \\ 1 & W_V \geq W_I \end{cases} \tag{1}$$

This is simply because if $W_V < W_I$, only a fraction of the interference affects the victim receiver; the rest is rejected by the IF filtering in the receive chain. More precisely, if $S_I(f)$ is the power spectral density (PSD)[27] of the interfering signal at the victim receiver and $H_V(f)$ is the effective bandpass characteristic (frequency selectivity), including the effect of all the IF filtering, then the effective average interference power seen by the victim receiver is

$$I = \int_{-\infty}^{\infty} |H(f)|^2 S_I(f) df \ . \tag{2}$$

The bandwidth correction factor (BCF) is a special case of the "interference correction factor" (ICF), which accounts for more than just the bandwidth ratio. In analysis of potential implementations of the ITemp concept, the ICF is needed to accurately reflect not only the average power received from an interference source (within the victim receiver's passband), but the effect of the interference. This is needed because not all interference waveforms effect victim receivers like Gaussian noise. Otherwise, the ICF would be the same as the BCF.

Deriving an ICF for a particular pairing of an interfering waveform and a victim receiver architecture can be complicated, but it is useful to establish a general definition that can be used in interference link budget calculations. The definition used here is as follows. Let $N_{V,pb}$ represent the average Gaussian noise power within the victim receiver passband corresponding to some threshold performance measure; e.g., a frame error rate (FER) of 0.01 in a digital receiver, or a baseband SNR in an analog receiver, given some

[27] The PSD is the average power per Hz as a function of frequency.

received desired signal power C_V. Let $\bar{I}_{V,pb}$ be the average received interference power within the victim receiver passband corresponding to the same performance. The "waveform sensitivity factor" is then:

$$WSF = \frac{\bar{I}_{V,pb}}{N_{V,pb}} \tag{3}$$

The interference correction factor is then

$$ICF = BCF \cdot WSF \tag{4}$$

Thus, the smaller the ICF, the less sensitive the victim receiver is to the interference. If $ICF = 0$, there is no interference.

As an example, assume that $ICF = -20\ \mathrm{dB}$ and the victim receiver requires that $C/N = 10\ \mathrm{dB}$ at the specified performance point. If the received desired signal power is –60 dBm, then the interference that can be tolerated is then –50 dBm (some of this may be outside the victim receiver passband).

If the interference appears noise-like to the victim receiver, then $WSF = 1$ and $ICF = BCF$. Generally, this is the case if the symbol rate of the interfering signal is substantially greater than the bandwidth of the victim receiver.

Background on Authors

Dr. Jay E. Padgett joined Telcordia Technologies in December 1999 as a Senior Research Scientist. Prior to that (since 1977), he was with Bell Laboratories, most recently part of Lucent Technologies. Since joining Telcordia, he has been involved with capacity and performance of 2G/3G/4G wireless network technologies, interference measurement and modeling, and ultra wide band (UWB) radio. From June 2002 until January 2004 he was the Principal Investigator for Telcordia's modeling and simulation work on the DARPA NETEX program, focusing on the effects of UWB transmissions on narrowband radios.

While at Bell Labs, he was most recently on the development team that created the DEFINITY® wireless business system (DWBS), a wireless PBX using the unlicensed PCS (UPCS) band. He developed a simulation and an analytical queuing model of DWBS Medium Access Control (MAC) layer operation to verify effects of different traffic load and handoff rates, and developed and implemented test procedures to verify compliance with the FCC "etiquette" for the UPCS band.

Dr. Padgett served as the President of the Wireless Information Networks Forum (WINForum, 1998-99), Chair of the TIA Wireless Consumer Communications Section (1993-99), and Editor of the American National Standard ANSI C63.17, which specifies test procedures for FCC certification of UPCS equipment. He received the Distinguished Technical Staff award from Bell Labs in 1986. He holds a B.S. and M.E. in Electrical Engineering from the University of Virginia, and a Ph.D. in E.E. from Polytechnic University, in Brooklyn, NY. He is a member of Tau Beta Pi, Eta Kappa Nu, and a Senior Member of the IEEE.

Dr. Robert A. Ziegler rejoined Telcordia Technologies in September 2003 as a Chief Scientist in the Wireless Systems & Networks Research Department. He is currently Program Manager for Telcordia's participation in the Lead Systems Integration effort for the US Army's Future Combat Systems program, in partnership with Boeing and SAIC. He was also with Telcordia (formerly Bell Communications Research) from 1991 to 2000, during which time he participated in, managed and led activities in algorithm research, system architecture, prototype development and technology transfer for low-power wireless and mobile communications.

From 2000 until 2003, Dr. Ziegler was Director of User Terminal Research and Development, and then Director of ASIC and User Terminal Product Management, for ArrayComm, Inc. There he continued his work in the themes of wireless system architecture, prototype development and technology transfer, this time around ArrayComm's wireless communication system concepts that incorporate its adaptive antenna array signal processing technology.

Dr. Ziegler holds a B.S. in Systems Science from Michigan State University, and a M.S. and Ph.D. in Electrical Engineering from Stanford University. He is a Senior Member of the IEEE.